MECHANICS.

66. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

The distance, parallel to the axis, from the mid-point of a chord to the arc of a parabola is constant. Show that the center of gravity of all segments formed by the chord is an equal parabola.

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, P. O., University, Miss.

Taking the tangent parallel to the chord as y-axis, and the corresponding diameter as x-axis, denoting the inclination of the axes by β, and the latus rectum by $2p$, the equation of the parabola is

$$y^2=\frac{2p}{\sin^2\beta}x.$$

Since all chords parallel to the base of the segment are bisected by the x-axis, the center of gravity is on this axis.

$\bar{x}$ being the abscissa of the center of gravity and a the distance from the origin to the mid-point of the chord, we have, by taking moments about the tangent,

$$\bar{x}\sin\beta=\frac{\int_0^a y\sin\beta dx.x\sin\beta}{\int_0^a y\sin\beta dx}, \text{ or } \bar{x}=\frac{\int_0^a \sqrt{(2p)}x^{\frac{3}{2}}\,dx}{\int_0^a \sqrt{(2p)}x^{\frac{1}{2}}\,dx}=\tfrac{3}{5}a.$$

Since a is constant for all positions of the chord, if the given parabola be moved to the right a distance equal to $\frac{3}{5}a$, it will coincide with the locus of the centers of gravity. Hence this locus is an equal parabola.

67. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A conical stick of timber, length a, radius of base r, and density δ, is depressed, apex downward, in a liquid, density δ', so that the base is just level with the liquid. If left free to rise, required the greatest altitude to which it will ascend.

I. Solution by WILLIAM HOOVER, A. M., Ph. D., Professor of Mathematics and Astronomy in Ohio State University, Athens, Ohio.

Let x=the part of the axis above the liquid at any time t from the beginning of the motion ; then the volume of the cone being $\frac{1}{3}\pi r^2 a$, the part under the liquid is $\frac{1}{3}\pi r^2 a[(a-x)/a]^3$, and if ρ be the density of the cone, the force acting on the cone is proportional to the members of the equation

$$\tfrac{1}{3}\pi r^2 a\,\rho\frac{vdv}{dx}=g[\tfrac{1}{3}\pi r^2 a\delta'\left(\frac{a-x}{a}\right)^3-\tfrac{1}{3}\pi r^2 a\rho] \quad\ldots\ldots\ldots\ldots\ldots\ldots(1),$$

v being the velocity.

$$\text{Or (1) is } a^3\rho\frac{vdv}{dx}=g[\delta'(a-x)^3-\rho a^3] \quad \ldots\ldots\ldots\ldots\ldots\ldots(2).$$

Integrating (2), $\frac{1}{2}a^3\rho v^2=g[-\frac{1}{4}\delta'(a-x)^4-\rho a^3x]+C$ ....(3).

When $x=0$, $v=0$, $\therefore$ $C=\frac{1}{4}(\delta' a^4 g)$ and (3) becomes

$$\tfrac{1}{2}(a^3\rho)v^2=g\{(\tfrac{1}{4}\delta')[a^4-(a-x)^4]-\rho a^3x\} \quad \ldots\ldots\ldots\ldots(4).$$

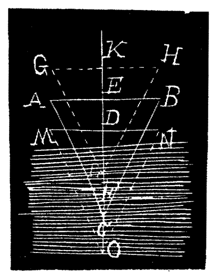
At the greatest height, $v=0$, and then

$$(\tfrac{1}{4}\delta')[a^4-(a-x)^4]-\rho a^3x=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(5),$$

a biquadratic for x. One value plainly being 0, there remains a cubic to be solved.

II. Solution by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Let $DK=x$, $DL=y$, $\therefore$ $x+y=a$. When the cone is under the liquid its acceleration is $g(\delta'-\delta)/\delta$.



The equation of motion is $d^2x/dt^2=\beta y^3-g$.

When $y=a$, $d^2x/dt^2=g(\delta'-\delta)/\delta$.

$\therefore$ $\beta=\delta' g/a^3\delta$.

$\therefore$ $d^2x/dt^2=(g/a^3\delta)(\delta' y^3-\delta a^3)=(g/a^3\delta)[\delta'(a-x)^3-\delta a^3]$.

$\therefore$ $(dx/dt)^2=-(g/2a^3\delta)[(a-x)^4\delta'+4a^3x\delta]+A$.

When $y=a$, $dx/dt=0$, $x=0$.

$\therefore$ $A=ag\delta'/2\delta$.

$\therefore$ $(dx/dt)^2=v^2=(ag\delta'/2\delta)-(g/2a^3\delta)[\delta'(a-x)^4+4\delta a^3x]$.

When $v=0$, $\delta'(a-x)^4+4\delta a^3x=a^4\delta'$, or

$$x^3-4ax^2+6a^2x=4a^3(\delta'-\delta)/\delta'.$$

This cubic determines x. When $\delta<\frac{1}{4}\delta'$, the cone leaps out of the liquid. At this moment $x=a$, $\therefore$ $v^2=ag(\delta'-4\delta)/2\delta$.

Then $h=v^2/2g=a(\delta'-4\delta)/4\delta=$height of vertex above water.

$\therefore$ Total height of base above liquid$=a+h$.

68. Proposed by G. B. M. ZERR, A. M., Ph. D., Professor of Mathematics and Science, Chester High School, Chester, Pa.

Find the horizontal and vertical components of the moon's "disturbing force" for any point on the earth's surface making an angle φ with the line joining the center of the earth to the center of the moon.

Solution by the PROPOSER.

Let CM, the distance from the earth to the moon$=d$; CA, the radius of the earth$=a$; O the point on the earth; $\angle OCA=\varphi$; m=moon's mass; $f=$ "moon's disturbing force" in direction MN; f'=disturbing force in direction ON.

Then $f'=(m/OM^2)\times(ON/OM)=m(ON/OM^3)$.

$ON=a\sin\varphi$, and we can take $OM=d$ without any appreciable error.